Chapter 2 - Day 1

Calculus describes how quantities change.

We are familiar with velocity (speed). If we drive 210 miles in 3 hours, then our average velocity/speed, or rate of travel is $210/3 = 70$ miles per hour

that is

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time elapsed}} = \frac{\Delta s}{\Delta t}$$

In general,

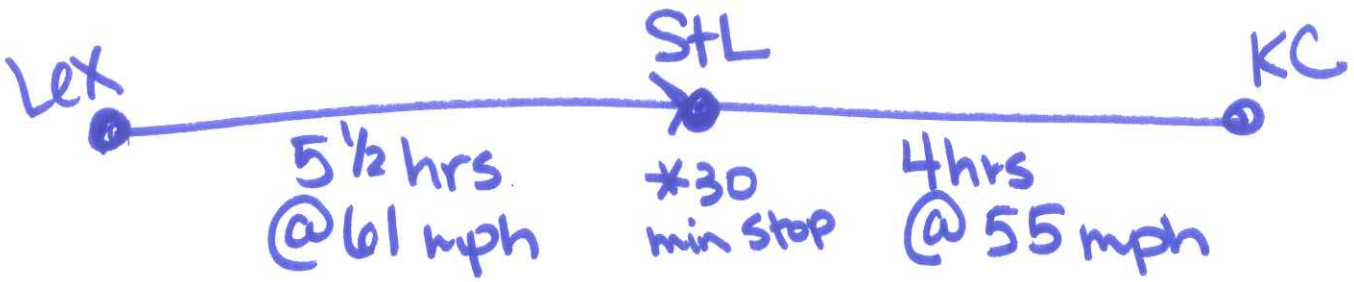
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{average rate of change of } y \text{ with respect to } x}{}$$

Ex: You drive to St. Louis from Lexington, which is a 337 mile trip. The drive takes you $5\frac{1}{2}$ hours. What is the average velocity of your car?

$$\text{average velocity} = \frac{337}{5.5} = 61.2\overline{72}$$

$$\approx 61 \text{ mph}$$

Ex: You drive to St. Louis in $5\frac{1}{2}$ hours averaging 61 mph. After a 30 minute snack break, you drive 4 hours averaging 55 mph on your way to Kansas City. How fast did your car average all day (include the stop)?



we need total time: $5.5 + .5 + 4 = 10$ hrs.

we need total distance: $d = r \cdot t$

distance = rate · time

$d = (61)(5.5) = 335.5$

$d = (55)(4) = 220$

} 555.5 total miles

$$\text{average velocity} = \frac{\text{miles}}{\text{time}} = \frac{555.5}{10} = 55.55 \text{ mph}$$

the <u>average rate of change</u> of a function $y = f(x)$ is:

$$\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

avg. rate of change is also the slope of the secant line between two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$

Ex: let $g(x) = 3 + 2(x-1)$. Find the average rate of change between $x=1$ and $x=4$.

$g(1) = 3 + 2(1-1) = 3 + 2(0) = 3$ $\quad (1,3)$

$g(4) = 3 + 2(4-1) = 3 + 2(3) = 9$ $\quad (4,9)$

avg. rate of change $= \frac{\Delta y}{\Delta x} = \frac{9-3}{4-1} = \frac{6}{3} = \boxed{2}$

* Since avg. rate of change is slope of secant line

$g(x) = 3 + 2(x-1) = 3 + 2x - 2 = \underline{\underline{2}}x + 1$

thus avg rate of change $= m = \boxed{2}$

Ex: Let $f(x)=\sqrt{2x+5}$. Find the average rate of change as x goes from -2 to 2

$f(-2)=\sqrt{2(-2)+5}=\sqrt{1}=1$ $\quad (-2,1)$

$f(2)=\sqrt{2(2)+5}=\sqrt{9}=3$ $\quad (2,3)$

$$AROC=\frac{\Delta y}{\Delta x}=\frac{3-1}{2--2}=\frac{2}{4}=\boxed{\frac{1}{2}}$$

Ex: Find the average velocity of a moving train whose position is given by $g(t)=4t^2+3t$ as t changes from 0 to 5 seconds.

$g(0)=4(0)^2+3(0)=0$

$g(5)=4(5)^2+3(5)=115$

$$AROC=\frac{115-0}{5-0}=\frac{115}{5}=\boxed{23}$$

Ex: Let $g(x)=\frac{1}{x}$. Find an x such that the AROC from 1 to x equals $\frac{-1}{10}$.

$g(1)=\frac{1}{1}=1$

$$AROC=\frac{\Delta y}{\Delta x}=\frac{g(x)-g(1)}{x-1}=\frac{\frac{1}{x}-1}{x-1}\cdot\frac{x}{x}$$

$$=\frac{1-x}{x(x-1)}=\frac{-1}{x}$$

we want $\frac{-1}{x}=\frac{-1}{10}$ so $\boxed{x=10}$